Tutorial 10

University of Victoria

CSC 320 - Spring 2023

Foundations of Computer Science

Teaching Team

Learning Outcomes:

- Become familiar with NP Membership.
- Review and understand verifiers and certificates.
- Write proofs using reduction.

Interesting Article:

"An Improved Exact Algorithm for Minimum Dominating Set in Chordal Graphs" [1]

March 28th, 2023

Question 10.01

NP-Completeness - Polynomial Time Reductions

Given an input $\langle G, k \rangle$ for Clique, where $G = (V, E)$ is an undirected graph and k is a positive integer. Use reduction to prove the following:

$$\text{Clique} \leq_{\text{p}} \text{Independent Set (IS)}$$

Question 10.02

NP-Completeness - Polynomial Time Reductions

Given an input $\langle G, k \rangle$ for IS, where $G = (V, E)$ is an undirected graph and k is a positive integer. Use reduction to prove the following:

$$\text{Independent Set (IS)} \leq_{\mathrm{p}} \text{Vertex Cover (VC)}$$

Question 10.03

NP-Completeness - Polynomial Time Reductions

Let $\langle G, k\rangle$, $G = (V, E)$ and $k \in \mathbb{N}$, be an instance for VC. Use reduction to prove the following:

$$\text{Vertex Cover (VC)} \leq_{\mathrm{p}} \text{Dominating Set (DS)}$$

for graphs without singletons.

Question 10.04

NP-Completeness - Polynomial Time Reductions

Given an input $\langle G, k \rangle$ for VC, where $G = (V, E)$ is an undirected graph and k is a positive integer. Use reduction to prove the following:

$$\text{Vertex Cover (VC)} \leq_{\mathrm{p}} \text{Independent Set (IS)}$$

Question 10.05

NP-Completeness - Polynomial Time Reductions

Given an input $\langle G, k \rangle$ for IS, where $G = (V, E)$ is an undirected graph and k is a positive integer. Use reduction to prove the following:

$$\text{Independent Set (IS)} \leq_{\mathrm{p}} \text{Clique}$$

Question 10.06

Membership in NP - Verifiers

We remember that Nondeterministic Polynomial Time (NP) is the set of languages for which there exists a polynomial time verifier. Thus, we can prove something is in NP by writing a verifier!

Verifiers

Verifiers take a problem and a potential solution C and check if C is actually a solution or not.

Consider the problem GraphColouring...

Question 10.07

Membership in NP - Verifiers

Prove CLIQUE is in NP.

$$\text{CLIQUE} = \{\langle G, k\rangle \mid G \text{ is a graph, } k \in \mathbb{Z}, k \geq 0.\}$$

where there is a clique of at least size k in G.

Question 10.08

Membership in NP - Verifiers

Prove INDSET is in NP.

$$\text{INDSET} = \{\langle G, k\rangle \mid G \text{ is a graph, } k \in \mathbb{Z}, k \geq 0.\}$$

where there is an independent set of at least k in G.

Bibliography

[1] F. N. Abu-Khzam, "An improved exact algorithm for minimum dominating set in chordal graphs," *Information Processing Letters*, vol. 174, p. 106 206, 2022, ISSN: 0020-0190. DOI: `https://doi.org/10.1016/j.ipl.2021.106206`. [Online]. Available: `https://www.sciencedirect.com/science/article/pii/S0020019021001216`.