Tutorial 09

University of Victoria

CSC 320 - Spring 2023

Foundations of Computer Science

Teaching Team

Learning Outcomes:

- Use reduction to prove a language is undecidable.
- Become familiar with reduction.

Interesting Article:

"Decidable and Undecidable Problems about Quantum Automata" [1]

March 21st, 2023

Question 9.01

Prove that the following language is undecidable by reduction from A_{TM}.

$$A_{TM} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$$

$$\text{Regular}_{TM} = \{\langle M\rangle \mid M \text{ is a TM and } L(M) \text{ is a regular language}\}$$

Question 9.02

Prove that the following language is undecidable by reduction from A_{TM}.

$$A_{TM} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$$

$$S_{TM} = \{\langle M\rangle \mid M \text{ is a TM that accepts } w^r \text{ whenever it accepts } w\}$$

Question 9.03

Prove that the following language is undecidable by reduction from A_{TM}.

$$A_{TM} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$$

$$S_{TM} = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \emptyset\}$$

Question 9.04

Prove that the following language is undecidable by reduction from A_{TM}.

$$A_{TM} = \{\langle M, w\rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$$

$$E_{TM} = \{\langle M\rangle \mid M \text{ is a TM and } L(M) = \emptyset\}$$

Question 9.05

Prove that the following language is undecidable by reduction from ALL_{CFG}.

$$ALL_{CFG} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$$

$$EQ_{CFG} = \{\langle G, H \rangle \mid G \text{ and } H \text{ are CFGs and } L(G) = L(H)\}$$

Bibliography

[1] V. D. Blondel, E. Jeandel, P. Koiran, and N. Portier, "Decidable and undecidable problems about quantum automata," *SIAM Journal on Computing*, vol. 34, no. 6, pp. 1464–1473, 2005. DOI: 10.1137/S0097539703425861. [Online]. Available: https://doi.org/10.1137/S0097539703425861.