University of Victoria

CSC 320 - Spring 2023

Foundations of Computer Science

Tutorial 07

Teaching Team

Learning Outcomes:

- Construct High Level Turing Machines.
- Distinguish between different Turing Machine variants.
- Become familiar with Deciders vs Looping.

Interesting Article:

"Note on A Universal Quantum Turing Machine" [1]
"Quantum Chaos in Quantum Turing Machines" [2]

March 7th, 2023

Question 03

Give a high level description of a Turing Machine which decides:

$$B = \{a^i b^j c^k \mid i \times j = k \text{ and } i, j, k \geq 1\}$$

We can start by building intuition. Let's see what is and isn't in B.

NOT: ε, abcc, aabc, b, c, a, ab.

IN: abc, aabbcccc, abbcc.

a^+ means at least one a.

T = "On input x:

1. Sweep across tape and check that input is of form $a^+b^+c^+$.
   If not, reject.
2. Move tape to first uncrossed 'a'. Cross it.
3. Alternate between crossing off one 'b' and then one 'c' until all 'b's are crossed.
   If all c's are crossed but there remain any uncrossed 'b's, reject.

   ex. aabbc, ✗abbc, ✗a✗bc, ✗a✗b✗, reject.
4. Uncross the crossed 'b's.
5. Repeat steps 2 and 3 until all a's are crossed
6. Sweep tape again and make sure all input symbols have been crossed.
   a) If so → accept.
   b) If not → reject." ∎

ex. aabbccc

← step 1 encounters one a, one b, and one c. ☺

aabbcccc, ✗abbcccc, ✗a✗bcccc, ✗a✗b✗ccc, ✗a✗✗✗ccc, ✗a✗✗✗✗cc.
✗abb✗✗cc, ✗✗bb✗✗cc, ✗✗✗b✗✗✗c, ✗✗✗✗✗✗✗c, ✗✗✗✗✗✗✗✗.

all a's crossed ☺

→ we sweep and everything is crossed ☺

Question 05

Give a high level description of a nondeterministic TM which recognizes the language:

$$L = \{1^n \mid n \text{ is a composite number}\}$$

Note: Composite number:
- 4 since 2×2
- 6 since 2×3
- 8 since 2×4
- 9 since 3×3
- ⋮

This statement is very important!

M = "On input x:

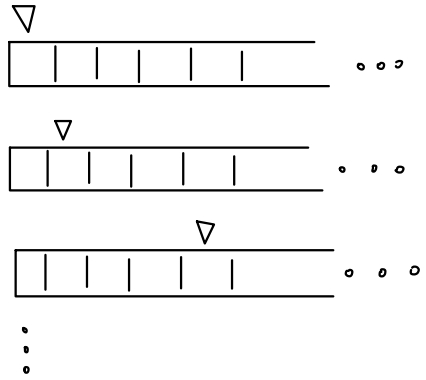
1. Sweep tape and make sure input is of form 1^*.
   If not, reject.
2. Nondeterministically choose numbers p and q $< n$.
   Rewrite tape to be

   $1^n \# 1^p \# 1^q$

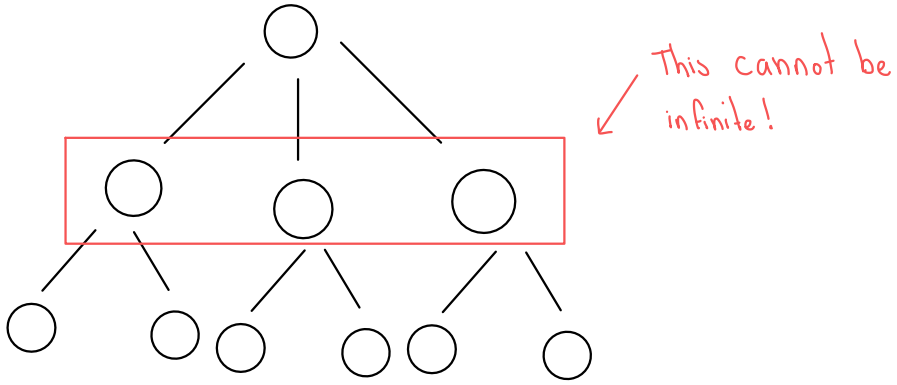
3. Check if $p \times q = n$ (We can use our TM for language B!)
   If yes, accept.
   If no, reject." ▲

What are the different TM variants? Give a brief explanation of each TM variant.

Multitape



Non deterministic



TM

Turing Machine where head can move $\{L, R, S\}$.

Enumerators

Outputs all strings of a language.

List all TM variants that are equivalent to the deterministic single tape TM.

- Multitape
- Nondeterministic TMs
- Enumerators

What is a Turing-Decidable language? What is the difference to a Turing-Recognizable language?

Turing-Decidable

On every input, always halts.

Turing-Recognizable

May enter an infinite loop.

Resources

[1] S. Iriyama, T. Miyadera, and M. Ohya, "Note on a universal quantum turing machine," English, *Physics letters. A*, vol. 372, no. 31, pp. 5120–5122, 2008, ISSN: 0375-9601.

[2] I. Kim and G. Mahler, "Quantum chaos in quantum turing machines," English, *Physics letters. A*, vol. 263, no. 4, pp. 268–273, 1999, ISSN: 0375-9601.