University of Victoria

CSC 320 - Spring 2023

Foundations of Computer Science

Tutorial 06

Teaching Team

Learning Outcomes:

- Become familiar with Context Free Languages.
- Use the Context Free Language Pumping Lemma.
- Gain understanding of High Level description of a Turing Machine.

Interesting Article:

"Regular Patterns, Regular Languages and Context-Free Languages" [1]

February 28th, 2023

Question 01

Show that the following language is not CFL using the CFL pumping lemma:

$$A = \{a^i b^j c^k \mid 0 \le i \le j \le k\}$$

We can begin by building our intuition. What is and isn't in the language?

NOT: aaabc, aaaabbc, aaabccccc,

IN: ε, abc, aabbcc, abbccc,

Suppose, for a contradiction, A is a CFL. Then by the CFL pumping lemma, there is a pumping length p such that for every string $s \in L$ where $|s| \ge p$, then $s = uvxyz$ where

1. $uv^i xy^i z \in L$ for each $i \ge 0$.
2. $vy \ne \varepsilon$.
3. $|vxy| \le p$.

Consider $w = a^p b^p c^p$, where $w \in L$ since it satisfies the constraints and $|w| \ge p$ since $p+p+p = 3p \ge p$. So, by the CFL pumping lemma, $w = uvxyz$ for some u, v, x, y, z where the above conditions hold.

There are two cases:

1. v and y each contain one type of symbol. Then it must be that one of the symbols a, b, c do not appear in v or y.
   i) If a does not appear, then uv^0xy^0z will contain the same number of a's but less b's or c's. So $uv^0xy^0z \notin L$.
   ii) If b does not appear, similarly to i) $uv^0xy^0z \notin L$.
   iii) If c does not appear, similarly to i) $uv^2xy^2z \notin L$.
2. When v or y contains more than one type of symbol, then uv^2xy^2z will have symbols out of order. So, $uv^2xy^2z \notin L$.

Therefore, by the CFL pumping lemma, A is not a CFL. ∎ ← Remember closing statement!

i) $\underbrace{a\ldots a}_{p}\underbrace{b\ldots b}_{p}\underbrace{c\ldots c}_{p}$ where v contains only b's and y only contains c's.

$u \quad \underbrace{v \quad x \quad y}_{p} \quad z$

② $\underbrace{a\ldots a}_{p}\underbrace{b\ldots b}_{p}\underbrace{c\ldots c}_{p}$

$u \quad \underbrace{v \quad x \quad y}_{p} \quad z$

where v contains both a's and b's, so if pump we will

From a a a a a b b b b b c c c c c
u v x y z

obtain a a a a ab ab ab ab ab b bbbb b ccccc where the string is not in L.

Question 02

Show that the following language is not CFL using the CFL pumping lemma:

$$B = \{0^n\#0^{2n}\#0^{3n} | n \geq 0\}$$

We start by identifying some strings that are and are not in the language.

NOT: 0#0#0, ε, 0#0#, #0#,

IN: 0#00#000, ##,

Suppose, for a contradiction, B is a CFL. Then by the CFL pumping lemma, there is a pumping length p such that for every string $s \in B$ where $|s| \geq p$, then $s = uvxyz$ where

1. $uv^ixy^iz \in B$ for each $i \geq 0$.
2. $vy \neq \varepsilon$
3. $|vxy| \leq p$

Important part of the proof! Do not omit.

Suppose $p = 2$ then

00#0000#000000

which is in the language.

Consider $w = 0^p\#0^{2p}\#0^{3p}$, then $w \in B$ and $|w| \geq p$, so by the CFL pumping lemma, $w = uvxyz$ for some u, v, x, y, z where the above conditions hold.

To avoid contradicting ①, we know that neither v nor y contains #, since otherwise uv^2xy^2z has at least 3 #'s so $uv^2xy^2z \notin B$.

$$\underbrace{0\ldots0}_{p}\#\underbrace{0\ldots0}_{2p}\#\underbrace{0\ldots0}_{3p}$$

u Vxy z

Where $v = \#0$, then pumping would result in

00#0000#000000

00#0#0000#000000

which would not be in B.

Only for visual learning, not in the proof!

Because of ③, at least one of $0^p, 0^{2p}, 0^{3p}$ have no symbols in v or y, so when we pump up to uv^2xy^2z, we cannot have a string of form $0^p\#0^{2p}\#0^{3p}$ since the ratio 1:2:3 of 0's will not hold in at least one place, so $uv^2xy^2z \notin B$.

$$\underbrace{0\ldots0}_{p}\#\underbrace{0\ldots0}_{2p}\#\underbrace{0\ldots0}_{3p}$$

u v x y z

Where $v = 0$, then pumping would result in

00#0000#000000

0 00#0000#000000

Which would not be in B.

Hence by the CFL pumping lemma, B is not a CFL. ∎

Question 03

Give a high level description of a Turing Machine which decides:

$$C = \{0^{2^n} \mid n \geq 0\}$$

<u>NOT</u>: ε, 000,

<u>IN</u>: 0, 00, 0000,

Hint: Review page 143 in the Textbook.

We begin by creating our Turing Machine M which decides C.

M = " On input string w:

1. Sweep left to right across tape, crossing off every other 0.
2. a) If there was only one 0 on the tape in step 1, accept.

   b) If there was an odd number (which wasn't 1) on the tape in step 1, reject.
3. Return head to left end of tape.
4. Go to step 1. " ∎

Remember this is an informal description!

This is a high level description:

- English prose to describe algorithm, ignoring implementation details.
- At this level: no need to mention how TM manages its tape or head.

<u>Format</u>

Input to TM: string

Objects other than strings as input must be represented as string.

Resources

[1] S. Jain, Y. S. Ong, and F. Stephan, "Regular patterns, regular languages and context-free languages," English, *Information processing letters*, vol. 110, no. 24, pp. 1114–1119, 2010, ISSN: 0020-0190.