Tutorial 05

University of Victoria

CSC 320 - Spring 2023

Foundations of Computer Science

Teaching Team

Learning Outcomes:

- Become familiar with Context Free Grammars.
- Convert a Context Free Grammar into Chomsky Normal Form.
- Use a Pushdown Automata to describe a language.

Interesting Article:

"A Formalisation of the Cocke-Younger-Kasami Algorithm" [1]

February 14th, 2023

Question 5.01

Consider the following language over $\Sigma = \{0, 1\}$, find a set of rules that defines a Context Free Grammar (CFG) that recognizes the language:

$$L_1 = \{0^n 1^n \mid n \geq 0\} \cup \{1^n 0^n \mid n \geq 1\}$$

Question 5.02

Consider the following language over $\Sigma = \{0, 1\}$, find a set of rules that defines a CFG that recognizes the language:

$$L_2 = \{w \mid w \text{ starts and ends with the same symbol}\}$$

Question 5.03

Consider the following language over $\Sigma = \{0, 1\}$, find a set of rules that defines a CFG that recognizes the language:

$$L_3 = \emptyset$$

Question 5.04

Consider the following language over $\Sigma = \{0, 1\}$, find a set of rules that defines a CFG that recognizes the language:

$$L_4 = \{w \mid w \text{ contains at least three 1s}\}$$

Question 5.05

Consider the following language over $\Sigma = \{0, 1\}$, find a set of rules that defines a Context Free Grammar (CFG) that recognizes the language:

$$L_5 = \{0^n 1^m \mid 2n \leq m \leq 3n\}$$

Question 5.06

Consider the following language over $\Sigma = \{0, 1\}$, create a parse tree and show sequence derivations for the following string: 000111

$$L_6 = \{0^n 1^n \mid n \geq 0\}$$

Question 5.07

Prove or Disprove: Every subset of a Context Free Language (CFL) is a regular language.

Question 5.08

Convert the following CFG into Chomsky Normal Form:

$$
\begin{aligned}
S &\longrightarrow ASB \\
A &\longrightarrow aAS \mid a \mid \epsilon \\
B &\longrightarrow SbS \mid A \mid bb
\end{aligned}
$$

Question 5.09

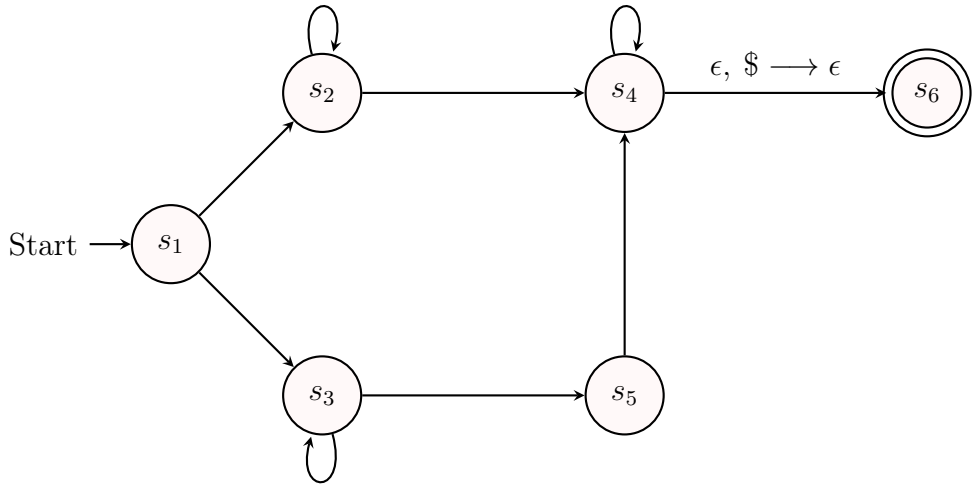
Convert the following CFG into Chomsky Normal Form:

$$
\begin{aligned}
S &\longrightarrow aXbX \\
X &\longrightarrow aY \mid bY \mid \epsilon \\
B &\longrightarrow X \mid c
\end{aligned}
$$

Question 10

Complete the state diagram by adding missing transitions so that it describes a PDA that recognizes the following language:

$$L = \{a^m b^n \mid m, n \geq 0 \text{ and (either } m = n \text{ or } m = n + 2)\}$$



where the PDA can be describe by the following:

State	Input Symbol	Stack Symbol	Next State	Stack Operation
s_1			s_2	
s_1			s_3	
s_2			s_2	
s_2			s_4	
s_3			s_3	
s_3			s_5	
s_4			s_4	
s_4	ϵ	\$	s_6	ϵ
s_5			s_4	

Table 3: PDA - Transition Table

Question 11

Derive or generate the string "aabaa" for the following grammar:

$$S \longrightarrow aAS \mid aSS \mid \epsilon$$
$$A \longrightarrow SbA \mid ba$$

Question 12

Convert the following CFG into Chomsky Normal Form:

$$S \longrightarrow AAA \mid \epsilon$$
$$A \longrightarrow aa \mid Aa \mid \epsilon$$

Bibliography

[1] M. Bortin, "A Formalisation of the Cocke-Younger-Kasami Algorithm," *Archive of Formal Proofs*, 2016, `https://isa-afp.org/entries/CYK.html`, Formal proof development, ISSN: 2150-914x.