CSC 370

Activity Worksheet: Projecting Functional Dependencies

Dr. Sean Chester

Fall 2022

Notes

This worksheet should provide extra practice questions for calculating a minimal basis of functional dependencies that apply to a projection of a relation. In each question you are given a relation, a projection of that relation, and a set of functional dependencies; you should show your work to project the functional dependencies onto the sub-relation. The first question is answered as a model solution.

Questions

1. R(A, B, C, D)

$$S = \pi_{(B,C,D)}(R)$$

A → B
B → C
C → D
D → A

Solution:

Observe first that all four attributes are keys, i.e., $\{A\}^+ = \{B\}^+ = \{C\}^+ = \{D\}^+ = \{A, B, C, D\}$.

To find FD's for S, we go through all subsets of $\{B, C, D\}$, compute their closures with respect to R, and restrict those closures to attributes of S. This yields: $\{B\}^+ = \{C\}^+ = \{D\}^+ = \{B, C, D\}$.

We would also find the same thing for non-singleton antecedents: $\{B, C\}^+ = \{B, D\}^+ = \{C, D\}^+ = \{B, C, D\}^+ = \{B, C, D\}$.

Thus, we construct the following non-trivial functional dependencies:

B → CD
C → BD
D → BC
BC → D
BD → C
CD → B

As we will learn in the next lesson, many of these FD's are redundant. We can simplify this to the following minimal basis by applying the splitting rule to the first three FD's and eliminating as many FD's as possible without changing the basis:

B → C
C → D
D → B

2. R(A, B, C)

$$S = \pi_{(B,C)}(R)$$

A → BC
B → A
C → A

Solution:

3. R(A, B, C, D)

$$S = \pi_{(B,C)}(R)$$

A → BC
BC → A
D → BC

Solution:

4. R(A, B, C, D)

$$S = \pi_{(B,C,D)}(R)$$

A → BC
BC → A
D → BC

Solution:

Solutions

Question 1

Observe first that all four attributes are keys, i.e., $\{A\}^+ = \{B\}^+ = \{C\}^+ = \{D\}^+ = \{A, B, C, D\}$.

To find FD's for S, we go through all subsets of $\{B, C, D\}$, compute their closures with respect to R, and restrict those closures to attributes of S. This yields: $\{B\}^+ = \{C\}^+ = \{D\}^+ = \{B, C, D\}$.

We would also find the same thing for non-singleton antecedents: $\{B, C\}^+ = \{B, D\}^+ = \{C, D\}^+ = \{B, C, D\}^+ = \{B, C, D\}$.

Thus, we construct the following non-trivial functional dependencies:

B $\rightarrow$ CD
C $\rightarrow$ BD
D $\rightarrow$ BC
BC $\rightarrow$ D
BD $\rightarrow$ C
CD $\rightarrow$ B

As we will learn in the next lesson, many of these FD's are redundant. We can simplify this to the following minimal basis by applying the splitting rule to the first three FD's and eliminating as many FD's as possible without changing the basis:

B $\rightarrow$ C
C $\rightarrow$ D
D $\rightarrow$ B

Question 2

This question is a bit easier, because there are only two possible non-trivial FD's on a relation S with two attributes:

$B \rightarrow C$
$C \rightarrow B$

We need only determine which, if any, of those follow from the original set of FD's.

Re: $B \rightarrow C$
Observe that:
$B \rightarrow A$ and
$A \rightarrow C$ (using the splitting rule)
$\therefore B \rightarrow C$

Re: $C \rightarrow B$
Observe that:
$C \rightarrow A$ and
$A \rightarrow B$ (using the splitting rule)
$\therefore C \rightarrow B$

Summary:
The set of FD's for the projection S are:
$B \rightarrow C$
$C \rightarrow B$

Question 3

As in question 2, this question is a bit easier, because there are only two possible non- trivial FD's on a relation S with two attributes:

B → C
C → B

We need only determine which, if any, of those follow from the original set of FD's.

Re: B → C
Observe that:
$\{B\}^+ = \{B\}$
∴ this FD does not follow.

Re: C → B
Observe that:
$\{C\}^+ = \{C\}$
∴ this FD does not follow.

Summary: Thus, the are no non-trivial FD's on S.

Question 4

Let us begin by calculating the closure of each subset of S with respect to the original FD's:

$\cancel{\{B\}^+ = \{B\}}$
$\cancel{\{C\}^+ = \{C\}}$
$\{D\}^+ = \{A, B, C, D\}$
$\{B, C\}^+ = \{A, B, C\}$
$\cancel{\{B, D\}^+ = \{A, B, C, D\}}$
$\cancel{\{C, D\}^+ = \{A, B, C, D\}}$

Note that I have struck-through any strict supersets of keys, since these can only produce redundant FD's. I have also struck-through any attribute sets which are equal to their own closures, as they cannot produce any non-trivial FD's.

Next, we can intersect the remaining examples with the set of attributes in S to obtain:

$\{D\}^+ = \{B, C, D\}$
$\{B, C\}^+ = \{B, C\}$

Now we can produce non-trivial FD's by observing for each X^+, what is in $X^+ \setminus X$.

D → BC

This is already a minimal basis; so, we have finished. Observe that this FD was already present in our original set.

Summary: The only FD in S is D → BC